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A PARALLEL COMPUTATION MODEL FOR NONLINEAR ELECTROMAGNETIC FIELD ANALYSIS BY HARMONIC BALANCE FINITE ELEMENT METHOD

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ABSTRACT

This paper presents a parallel computation model for the time-periodic nonlinear electromagnetic field analysis in the frequency domain using Harmonic Balance Finite Element Method(HBFEM). The proposed model, different from the traditional HBFEM technique that requires large memory and long CPU time, divides the global system matrix into a number of matrices in the frequency domain. Each computation unit has exactly the same number of elements and unknown values. The work involved in calculating the element matrices is equal, therefore the load can be well-balanced and the maximum speed-up will be M times if M processors are available (M is the number of harmonics considered in the electromagnetic field). The model is well-suited to MIMD parallel computer or multiple computers connected by Local Area Networks.

1 INTRODUCTION

Harmonic balance techniques have been widely used in solving nonlinear circuit problems in 1980's[1]. It was first introduced to the nonlinear quasi-static electromagnetic field analysis at the end of 1980's[2]. The HBFEM differs from traditional FEM time-domain methods, transient analysis and other time harmonic methods. The harmonic balance uses a linear combination of sinusoids to build the solution and uses the coefficients of the sinusoids to represent waveforms. It is combined with the finite element method to solve time-periodic steady-state nonlinear electromagnetic field problems. The HBFEM directly solves the steady-state problem of the electromagnetic field in the frequency domain and thus is often considerably more efficient than traditional time-domain methods when fields exhibit widely separated time constants and mildly nonlinear behavior.

Since practical application models always have a large dimension and complex structures and may consist of a large number of harmonics, an extremely large amount of calculations will be involved. The users are always suffered with the computer memory and CPU time to gain the accurate results. Various computing techniques such as block matrix method have been used to reduce memory by extending CPU time[3], but many real problems are still limited by computer power.

This paper discusses a parallel computation model for the time-periodic nonlinear electromagnetic field problems in the frequency domain by using

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HBFEM. The proposed model differs from the other parallel models in which the problem is divided into a large number of subdomains and separate calculations are performed on each subdomain[4, 5]. The basic concept of the proposed parallel model is to divide the global system matrix into frequency domain matrices and to map the harmonic components to the computation units. Each computation unit has exactly equal number of elements and unknown values. The work involved in calculating the element matrices is equal, therefore the load can be well-balanced and the maximum speed-up will be M times if M processors are available (M is the number of harmonics considered in the EM field).

2 CONCEPT OF HBFEM IN NON-LINEAR EM FIELDS

2.1 Basic Concept of Harmonic Balance Techniques

When a linear system of electromagnetic fields is excited by a sinusoid, its steady-state response is sinusoidal at the same frequency as the input. A nonlinear systems however will produce a lot of harmonics whose frequences are the integer times of the frequency of the input signal. The response can be represented by Fourier series. To solve such a nonlinear system, a linear combination of sinusoids is used to build the solution, which is in the frequency domain, using the coefficients of the sinusoids to represents waveforms. This is the so called harmonic balance technique.

2.2 HBFEM in Electomagnetic Fields

Combined with the finite element method, the harmonic balance technique directly solves the steady-state problem of the electromagnetic field in the frequency domain and thus is often considerably more efficient than traditional time-domain methods when fields exhibit widely separated time constants and mildly nonlinear behavior. If the steady-state response consists of just a few dominant sinusoids, as in common cases, the harmonic balance needs only a small data set to represent the response accurately. The advantage of using sinusoids to approximate a quasi periodic steady-state response becomes particularly important when the response contains dominant sinusoids at widely separated frequencies.

The electromagnetic field in either linear or nonlinear system satisfies Maxwell's equations. Considering a nonlinear magnetic system excited by current source J will give the potential equation of the electromagnetic field as follows:

$$\nabla \times \nu \nabla \times A + \sigma\left(\frac{\partial A}{\partial t} + \nabla \varphi\right) - J = 0 \qquad (1)$$

where ν is the magnetic reluctivity, σ the conductivity, A the vector potential, φ the scalar potential, J, the current density. For the ith node of the finite element, the potentials and the current density can be expressed using the assumption of the trigonometric functions[6]:

$$A^i = A_0^i + \sum_{k=1}^{\infty}\{A_{ks}^i \sin(k\omega t) + A_{kc}^i \cos(k\omega t)\} \qquad (2)$$

$$\varphi^i = \varphi_0^i + \sum_{k=1}^{\infty}\{\varphi_{ks}^i \sin(k\omega t) + \varphi_{kc}^i \cos(k\omega t)\} \qquad (3)$$

$$J = J_0 + \sum_{k=1}^{\infty}\{J_{ks} \sin(k\omega t) + J_{kc} \cos(k\omega t)\} \qquad (4)$$

where ks and kc indicate the kth sin and cos coefficients respectively.

If the properties of the nonlinear material are isotopic and only variable with electromagnetic fields, the response of nonlinear materials can be also expressed by an assumption of the trigonometric functions. The magnetic reluctivity can be expressed as[6]:

$$\nu(B(t)) = \frac{H\{B(t)\}}{B(t)} = \nu_0 + \sum_{k=2n-2}^{\infty}\{\nu_{ks} \sin(k\omega t) + \nu_{kc} \cos(k\omega t)\} \qquad (5)$$

where the Fourier coefficients are calculated by

$$\nu_0 = \frac{1}{T}\int_0^T \nu(t)dt \qquad (6)$$

$$\nu_{NS} = \frac{2}{T}\int_0^T \nu(t) \cdot \sin(n\omega t)dt \qquad (7)$$

$$\nu_{Nc} = \frac{2}{T}\int_0^T \nu(t) \cdot \cos(n\omega t)dt \qquad (8)$$

If the hysteresis characteristic of a magnetic core is considered, the approximate expression can be used to describe magnetization and hysteresis loop [7], that is

$$H\{B(t)\} = H_0(B) + H_e \frac{dB}{dt} = (\alpha B + \beta B^{2n-1}) + (\frac{1}{f}\frac{dB}{dt})\frac{dB}{dt} \tag{9}$$

where the first term indicates the saturation characteristic of the magnetic core, and the second term expresses the hysteresis characteristic, and α and β are the saturation coefficients, which can be obtained from the B-H curve of the magnetic core, and f is the excitation frequency.

Because the trigonometric function has an orthogonal characteristic, the harmonic potential A_k on each node of boundary also satisfies Dirichlet and Neumann boundary conditions, which are expressed in the frequency domain as follows:

$$A_k = \{A_0,\ A_{1s},\ A_{1c},\ A_{2s},\ A_{2c},\ \cdots\}^T \tag{10}$$

$$\frac{\partial A_k}{\partial n} = \{\frac{\partial A_0}{\partial n}, \frac{\partial A_{1s}}{\partial n}, \frac{\partial A_{1c}}{\partial n}, \frac{\partial A_{2s}}{\partial n}, \frac{\partial A_{2c}}{\partial n}, \ldots\}^T \tag{11}$$

2.3 System Matrix Equation of HBFEM[8]

Using Galerkin method and weighted function, Eq.(1) for two-dimensional problems in which φ is assumed to be zero can be written as follows:

$$\begin{aligned} G &= \iint_S \{\frac{\partial N_i}{\partial x}\nu\frac{\partial A}{\partial x} + \frac{\partial N_i}{\partial y}\nu\frac{\partial A}{\partial y}\}\, dxdy \\ &\quad - \iint_S \{J_s - \sigma\frac{\partial A}{\partial t}\} N_i dxdy = 0 \end{aligned} \tag{12}$$

Substitution of Eqs.(2) and (4) into Eq.(12) can obtain the following harmonic balance matrix equation for each element, in which only odd harmonics are considered and the orthogonal characteristic of trigonometric functions and harmonic balance techniques have been used:

$$\begin{aligned} &\frac{1}{4\Delta^e}\begin{bmatrix} (c_1c_1+d_1d_1)D & (c_1c_2+d_1d_2)D & (c_1c_3+d_1d_3)D \\ (c_2c_1+d_2d_1)D & (c_2c_2+d_2d_2)D & (c_2c_3+d_2d_3)D \\ (c_3c_1+d_3d_1)D & (c_3c_2+d_3d_2)D & (c_3c_3+d_3d_3)D \end{bmatrix}\{A_k\}^e \\ &+\frac{\sigma\omega\Delta^e}{12}\begin{bmatrix} 2N & N & N \\ N & 2N & N \\ N & N & 2N \end{bmatrix}\{A_k\}^e - \{K\}^e = 0 \end{aligned} \tag{13}$$

where

$$D = \frac{1}{2}\left[\begin{array}{cc|cc|c} 2\nu_0 - \nu_{2c} & \nu_{2s} & \nu_{2c} - \nu_{4c} & -\nu_{2s} + \nu_{4s} & \cdots \\ \nu_{2s} & 2\nu_0 + \nu_{2c} & \nu_{2s} + \nu_{4s} & \nu_{2c} + \nu_{4c} & \cdots \\ \hline & & 2\nu_0 + \nu_{6c} & \nu_{6s} & \cdots \\ & Symmetry & \nu_{6s} & 2\nu_0 - \nu_{10c} & \cdots \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right] \tag{14}$$

$$N = \left[\begin{array}{cc|cc|cc|c} 0 & -1 & 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \hline & & 0 & -3 & 0 & 0 & \cdots \\ & & 3 & 0 & 0 & 0 & \cdots \\ \hline & & & & 0 & -5 & \cdots \\ & & & & 5 & 0 & \cdots \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right] \tag{15}$$

$$\begin{aligned} \{K_i\}^e = \frac{\Delta^e}{3}\{&J_{1s}\ J_{1c}\ J_{3s}\ J_{3c}\ \ldots, \\ &J_{1s}\ J_{1c}\ J_{3s}\ J_{3c}\ \ldots, \\ &J_{1s}\ J_{1c}\ J_{3s}\ J_{3c}\ \ldots\}^T \end{aligned} \tag{16}$$

$$\begin{aligned} \{A_k\}^e = \{&A_{1s}^1\ A_{1c}^1\ A_{3s}^1\ A_{3c}^1\ A_{5s}^1\ A_{5c}^1\ \ldots, \\ &A_{1s}^2\ A_{1c}^2\ A_{3s}^2\ A_{3c}^2\ A_{5s}^2\ A_{5c}^2\ \ldots, \\ &A_{1s}^3\ A_{1c}^3\ A_{3s}^3\ A_{3c}^3\ A_{5s}^3\ A_{5c}^3\ \ldots\}^T \end{aligned} \tag{17}$$

in which the vector potential $\{A_k\}^e$ is called the frequency-domain vector potential, Δ^e is the area of the element, e is the element number, ω is the fundamental frequency, and c and d are obtained from x and y coordinates, $c_{ie} = y_{je} - y_{ke}$, $d_{ie} = x_{ke} - x_{je}$.

Since all harmonics are included in the calculation, the size of the matrix is *2M* larger than that of the traditional FEM. Fig.1 shows the comparison between the dimension of the system matrices of traditional FEM and HBFEM, where N is the number of unknown variables, M is the number of harmonics. Suppose K is the bandwidth of the matrix for traditional FEM, then the bandwidth of the HBFEM matrix is *2MK*.

Several practical application problems have been calculated by HBFEM on Fujitsu FCOM-M760/20 supercomputer. Limited by the computer memory and calculation ability, the maximum number of the unknowns was only 1200×3, in which only 2000 elements and 3 harmonic components were considered. To obtain accurate results, more elements and more harmonic components are required in most cases.

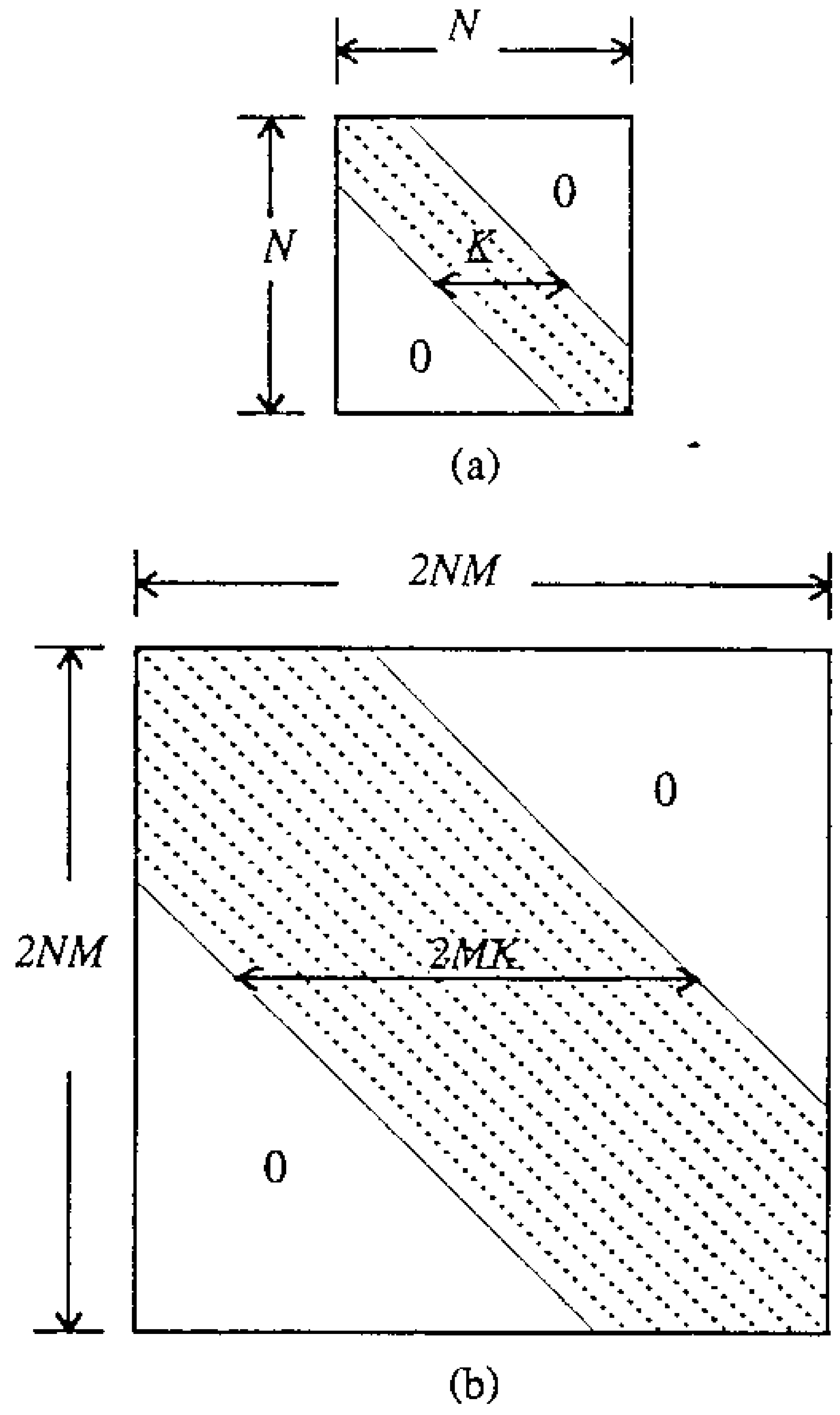

Fig.1 Comparison between the dimension of the system matrices of (a) traditional FEM and (b) HBFEM.

3 A PARALLEL COMPUTATION MODEL

To solve Eqs.(13) by parallel computation, we separate calculations by dividing the problem into each harmonics instead of dividing it into each finite element.

The matrices D and N can be represented by means of the submatrices:

$$D=\begin{bmatrix} D_{11} & D_{13} & D_{15} & \dots \\ D_{31} & D_{33} & D_{35} & \dots \\ D_{51} & D_{53} & D_{55} & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \tag{18}$$

$$N=\begin{bmatrix} N_1 & 0 & 0 & \dots \\ 0 & N_3 & 0 & \dots \\ 0 & 0 & N_5 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix} \tag{19}$$

where

$$D_{hh}=\frac{1}{2}\begin{bmatrix} 2\nu_0-\nu_{2hc} & \nu_{2hs} \\ \nu_{2hs} & 2\nu_0+\nu_{2hc} \end{bmatrix} \tag{20a}$$

$$D_{h(h+2p)}=\frac{1}{2}\begin{bmatrix} \nu_{2pc}-\nu_{2(h+p)c} & -\nu_{2ps}+\nu_{2(h+p)s} \\ \nu_{2ps}+\nu_{2(h+p)s} & \nu_{2pc}+\nu_{2(h+p)c} \end{bmatrix} \tag{20b}$$

$$N_h=\begin{bmatrix} 0 & -h \\ h & 0 \end{bmatrix}=h\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{21}$$

$$h=1,3,5,\dots$$
$$p=1,2,3,\dots$$

The vectors $\{A_k^e\}$ and $\{K_k^e\}$ can be re-arranged according to harmonics as followings

$$\begin{aligned}\{A_k\}^e &= \{A^1_{1s}\ A^1_{1c}\ A^2_{1s}\ A^2_{1c}\ A^3_{1s}\ A^3_{1c} \\ &\quad\ A^1_{3s}\ A^1_{3c}\ A^2_{3s}\ A^2_{3c}\ A^3_{3s}\ A^3_{3c} \\ &\quad\ \dots\}^T \\ &= \begin{Bmatrix} A_1 \\ A_3 \\ A_5 \\ \vdots \end{Bmatrix} = \{A_h\}^e\end{aligned} \tag{22}$$

$$\begin{aligned}\{K_k\}^e &= \{K_{1s}\ K_{1c}\ K_{1s}\ K_{1c}\ K_{1s}\ K_{1c} \\ &\quad\ K_{3s}\ K_{3c}\ K_{3s}\ K_{3c}\ K_{3s}\ K_{3c} \\ &\quad\ \dots\}^T \\ &= \begin{Bmatrix} K_1 \\ K_3 \\ K_5 \\ \vdots \end{Bmatrix} = \{K_h\}^e\end{aligned} \tag{23}$$

Then the matrix equation representing in harmonics is obtained:

$$[H]^e\{A_h\}^e=\{K_h\}^e \tag{24}$$

where

$$[H]^e = \begin{bmatrix} [H_{11}]^e & [H_{13}]^e & [H_{15}]^e & \dots \\ & [H_{33}]^e & [H_{35}]^e & \dots \\ & Symmetry & [H_{55}]^e & \dots \\ & & & \dots \end{bmatrix} \tag{25a}$$

$$[H_{hh}]^e = \frac{1}{4\Delta^e} \begin{bmatrix} (c_1c_1+d_1d_1)D_{hh} & (c_1c_2+d_1d_2)D_{hh} & (c_1c_3+d_1d_3)D_{hh} \\ (c_2c_1+d_2d_1)D_{hh} & (c_2c_2+d_2d_2)D_{hh} & (c_2c_3+d_2d_3)D_{hh} \\ (c_3c_1+d_3d_1)D_{hh} & (c_3c_2+d_3d_2)D_{hh} & (c_3c_3+d_3d_3)D_{hh} \end{bmatrix} + \frac{\sigma\omega\Delta^e}{12} \begin{bmatrix} 2N_h & N_h & N_h \\ N_h & 2N_h & N_h \\ N_h & N_h & 2N_h \end{bmatrix} \tag{25b}$$

$$[H_{h(h+2p)}]^e = \frac{1}{4\Delta^e} \begin{bmatrix} (c_1c_1+d_1d_1)D_{h(h+2p)} & (c_1c_2+d_1d_2)D_{h(h+2p)} & (c_1c_3+d_1d_3)D_{h(h+2p)} \\ (c_2c_1+d_2d_1)D_{h(h+2p)} & (c_2c_2+d_2d_2)D_{h(h+2p)} & (c_2c_3+d_2d_3)D_{h(h+2p)} \\ (c_3c_1+d_3d_1)D_{h(h+2p)} & (c_3c_2+d_3d_2)D_{h(h+2p)} & (c_3c_3+d_3d_3)D_{h(h+2p)} \end{bmatrix} \tag{25c}$$

By using the Gauss iteration method, the system equations for the fundamental component and the hth harmonics (h=3,5,7,...) are obtained as follows

$$\left.\begin{aligned} &[H_{11}]\{A_1\}^k = \{G_1\}^k \\ &[H_{33}]\{A_3\}^k = \{G_3\}^k \\ &\quad\vdots \\ &[H_{hh}]\{A_h\}^k = \{G_h\}^k \qquad (h=5,7,\dots) \end{aligned}\right\} \tag{26}$$

where

$$\left.\begin{aligned} &\{G_1\}^k = \sum_{j=3,5,\dots}[H_{1j}]\{A_j\}^{k-1} + \{K_1\} \\ &\{G_3\}^k = \sum_{j=1,5,\dots}[H_{3j}]\{A_j\}^{k-1} + \{K_3\} \\ &\quad\vdots \\ &\{G_h\}^k = \sum_{\substack{j=1,3,5,\dots \\ j\neq h}}[H_{hj}]\{A_j\}^{k-1} + \{K_h\} \quad (h=5,7,\dots) \end{aligned}\right\} \tag{27}$$

in which the superscripts k and k-1 denote the k and (k-1)th iterations.

It is noted that there is no superscript e in Eqs.(26) and (27) and it means that each equation for a given h is the system equation for the whole space region but a given harmonics. Compared to both the dimension of the system matrix of the traditional FEM and the dimension of the whole system matrix of HBFEM, the dimension of the system matrix of each harmonics in Eq.(26) is $2N\times 2N$ with the bandwidth of $2K$, where K is the bandwidth of the system matrix of the traditional FEM, as shown in Fig.1.

Eq.(26) for different harmonic components h can be solved by different computation unit. The parallel algorithm for each computation unit CU_h ($h \in \{1,3,5,\dots,2(M-1)+1\}$) is as follows:

1. Initialization
   (1) Calculate H_{hj} ($j=1,3,5,\dots,2(M-1)+1$)
   /*based on Eq.(25b) and (25c) */
   (2) Calculate K_h
   /*based on Eq.(23) */
   (3) Set A_h^0 to zero vector
   (4) Set i to 0, set k to 1
2. Repeat until $i \neq 0$
   (1) Broadcast A_h^{k-1} to all CU_h ($r \neq h$)
   (2) Receive A_r^{k-1} from all CU_h ($r \neq h$)
   (3) Calculate $\{G_h\}^k$
   /*based on Eq.(27) */
   (4) Solve $[H_{hh}]\{A_h\}^k = \{G_h\}^k$
   /*based on Eq.(26) */
   (5) Set k to $k+1$
   (6) Check convergence
   (7) If convergence then set i to 1

4 ANALYSIS OF THE MODEL

4.1 Parallelism

The model, as shown in the previous section, is a single programme multiple data (SPMD) model, in which each computation unit executes a copy of the same programme acting on its own data. In addition, thanks to the decomposition of the problem into harmonic components instead of into space regions, each computation unit has the same space structure of the finite elements and the same number of unknown variables and then the same structure of the matrices. The maximum parallelism is M, and the maximum speed-up will be M times if M processors are available.

4.2 The Amount of Computation of Each Computation Unit

It is implied in Eq.(26) that the whole system equation is solved by the Gauss iteration method and the matrix equation for each harmonic component is calculated by each computation unit. This iteration loop is referred to as the outer loop. In each computation unit, when solving Eq.(26) for one harmonic component in step 2(4), either iterative or direct solving method can be used. The iterative method is often less expensive in terms of memory and time than the direct method; however, the direct method is guaranteed to converge to a solution. No matter which one is used, the computation in step 2(4) is most expensive in CPU time in each iteration of the outer loop.

Suppose that N unknown potentials and M harmonic components are considered, then a linear equation set of order $2N$ is solved in step 2(4) in each of M computation units. Taking into account that H_{hh} and H_{hj} are symmetric sparse matrices with their bandwidth of $2K$, the amount of computation for solving Eq.(26) can be reduced. If the Gauss iteration method is employed in step 2(4), the numbers of floating point addition and floating point multiplication are both equal to $4NKL$, where L is the number of the inner iteration inside step 2(4). According to previous calculations in FCOM-M760/20 supercomputer[8], L is about 50 for guarantee of convergence. In addition, checking the convergence inside step 2(4) requires $2N$ floating point comparisons. Therefore, the time in step 2(4) is

$$T=L(4NKT_{add}+4NKT_{mul}+2NT_{comp}) \tag{28}$$

where T_{add}, T_{mul}, and T_{comp} are the time for a floating point addition, a floating point multiplication, and a floating point comparison, respectively.

Before Eq.(26) is solved in step 2(4), a vector $\{G_{hj}\}^k$ is prepared, requiring $4NK(M-1)$ floating point additions and floating point multiplications. Added with $2N$ comparisons in step 2(6) to check the whole system convergence, the total time for one iteration in the outer loop is

$$\begin{aligned} T_{cal}=&4NK(L+(M-1))T_{add} \\ &+4NK(L+(M-1))T_{mul}+2N(L+1)T_{comp} \end{aligned} \tag{29}$$

4.3 Communication and Synchronization

Although the computation unit spends most of the time on computations based on its own memory, its calculation result in each iteration in the outer loop is shared by all the other units. Step 2(1) in our model broadcasts the result, which is a message in the size of $2N$ floating point data, and then in step 2(2) $M-1$ messages in the same size will be received. If the hardware supports broadcasting, the communication cost is $2NMT_{comm}$, where T_{comm} is the time for broadcasting a datum as well as the time for receiving a datum. If the hardware does not support broadcasting, the point-to-point communication will be employed and the communication cost is $2NM(M-1)T_{comm}$.

The ratio of computation to communication on the computation unit in our model is therefore as follows,

$$\frac{Computation}{Communication}=\begin{cases} \dfrac{2K(L+(M-1))T_{add}+2K(L+(M-1))T_{mul}+(L+1)T_{comp}}{MT_{comm}} & (30a) \\[2ex] \dfrac{2K(L+(M-1))T_{add}+2K(L+(M-1))T_{mul}+(L+1)T_{comp}}{M(M-1)T_{comm}} & (30b) \end{cases}$$

where Eqs.(30a) and (30b) are for broadcasting and for point-to-point communication, respectively. The time for an addition, a multiplication, a comparison, and a communication, depends strongly on computer systems. In general, $T_{comm}>T_{mul}>T_{add}\approx T_{comp}$. For a particular case where N=1200, $K\approx N/2$=600, M=7, $L\approx$50, the ratio of computation to communication is roughly $4KLT_{add}/MT_{comm}\approx 17000T_{add}/T_{comm}$ for broadcasting or $4KLT_{add}/M^2T_{comm}\approx 2400T_{add}/T_{comm}$ for point-to-point communication.

It can be seen from the high ratio of computation to communication that our model is coarse-grained, which is well-suited to the MIMD parallel supercomputer such as POWERparallel System SP2[9] or multiple computer connected by Local Area Networks[10, 11].

Synchronization is automatically achieved in step 2(2) by using a blocking communication primitive *receive*, which returns only when messages from all CU_r ($r\neq h$) have arrived. To avoid wasting CPU cycles in synchronization phase, the same number of iterations is given to the inner iterations in the inner loop to balance the amount of computation in each computation unit. Doing so does not cause the whole system to diverge.

4.4 Memory Requirement

To reduce the calculation time in step 2(4), all H_{hj} ($j=1, 3, 5, ..$) are pre-calculated in steps 1(1) and 1(2) and then are stored in each computation unit. As mentioned above, for a practical case of N unknown variables, each H_{hj} is a symmetric sparse matrix with the bandwidth of $2K$. Each unit has M number of H_{hk} and then needs $2NKM$ floating point memory to store them. Together with the memory used by the $2N$ unknown variables and $2N$ temporary variables, the total memory of $2N(KM+2)$ is needed in each computation unit if the Gauss iteration method is employed in step 2(4).

It can be seen from Fig.1 that the memory for the system matrix of HBFEM is $2NM\times MK$. Taking into account the memory used by the $2NM$ unknown variables and $2NM$ temporary variables, the memory for HBFEM before parallelization is $2NM(KM+2)$. Therefore, the memory for each computation unit in our model is reduced to $1/M$.

5 CONCLUSION AND FUTURE WORK

A parallel computation model for nonlinear EM field analysis using HBFEM has been proposed in this paper. The model can be easily mapped on distributed-memory MIMD parallel computer systems or multiple computers connected by LANs. The future work includes implementation of the model in IBM POWERparallel System SP2 and in an Amoeba-based high performance distributed computing environment[10, 11] and analysis of nonlinear electromagnetic fields in magnetic components of switch-mode power supplies and in microwave integrated circuits. We intend to map the subdomains of the finite elements as well as the harmonic components to the computation units when the number of the harmonics considered is less than the number of the processors available.

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